Module 3

Neutron Cross-Sections, Neutron Density and Neutron Flux

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3.1 MODULE OVERVIEW

Microscopic cross-section

The purpose of this module is to introduce three key concepts which provide the basis for calculating ***neutron reaction rates*** in various components of the reactor. The first is the ***microscopic*** reaction cross-section. This parameter is a quantitative measure of the probability of a neutron undergoing some specified reaction with an ***individual nucleus*** of any one of the materials present in the reactor. The microscopic cross-section is important in deciding what materials may or may not be suitable for use in the reactor. We will also consider how the microscopic cross-section varies with the energy of the neutrons.

Macroscopic cross-section

While the microscopic cross-section conveys information about the probabilities of scattering, absorption or fission for an individual nucleus, the calculation of the rates of these processes for the reactor as a whole involves, in addition, the numbers of atoms of each material per unit volume of the reactor. Combining this number with the microscopic cross-section yields the **macroscopic** cross-section. The actual reaction rate is determined by multiplying the macroscopic cross-section by the third of our key concepts, the ***neutron flux.***

Once we understand these concepts, we can work out, for example, the reactor power level which corresponds to a given neutron flux. These concepts will also be used in many of the later modules, for example, in working out xenon poisoning and the rate of U-235 burnup and Pu-239 growth.

3.2 MODULE OBJECTIVES

After studying this module, you should be able to:

i) List the various reactions that a neutron can undergo during its lifetime.

ii) Define the **microscopic reaction cross-section** (including the unit used).

iii) Define the **macroscopic cross-section** (and its unit).

iv) Define the **neutron flux** and its use in determining the reaction rate per unit volume.

v) Describe in general terms the variation of absorption cross-section of U-235 and U-238 with neutron energy.

3.3 THE MICROSCOPIC CROSS-SECTION

Possible neutron reactions

Let us have a look at the various reactions a neutron can undergo with a U-235 nucleus:

a) As mentioned in Section 2.5, if the neutron energy is greater than 14 keV, ***inelastic scattering*** may occur. If the neutron energy is lower, there is no chance of this reaction happening;

b) The neutron may just emerge with a changed kinetic energy (***elastic scattering***) and this can happen at any neutron energy;

c) The neutron may be captured (***radiative capture***);

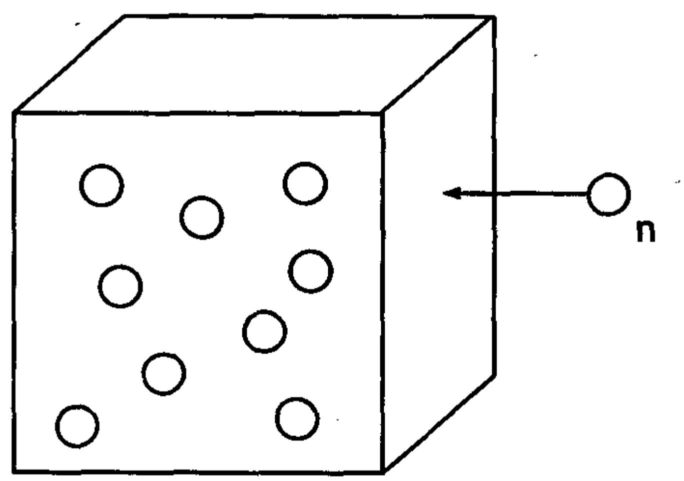
d) The neutron may cause ***fission***.

Radiative capture and fission are much more likely for slow neutrons than for fast neutrons, and fission in U-235 is always more probable than radiative capture for thermal neutrons.

Nuclear cross-section

In estimating what will happen to a neutron travelling in a reactor, we are always comparing the ***relative probabilities*** of the various reactions that it can undergo with the nuclei of the atoms present. We need some way to visualize what is going on and also to measure the probability of each reaction. This is done by introducing a quantity known as the ***nuclear cross-section.***

As an analogy, suppose that we have a large, thin-walled box, inside which a spherical balloon is floating. Imagine that you are now going to fire a bullet at random into the closed box. There is no way you can determine whether or not the bullet will hit the balloon. On the other hand, if we know the sizes of the balloon and the box, we can easily calculate the ***probability*** that a randomly fired bullet will hit it. We can visualize the balloon as presenting a certain "target area" to incoming bullets; a balloon of radius r will appear as a flat disc of area πr^2. The probability of the bullet hitting the balloon is directly proportional to this area, which we call the "cross-section" of the target.



Let's extend this concept to the problem of estimating the probability of neutrons interacting with nuclei. Consider a neutron fired into a volume containing a certain number of nuclei of the material whose cross-section we want to define. For the balloon, the cross-section which determined the probability of a hit was simply the physical target area which it presented to the bullet. The neutron, however, can undergo a number of different reactions with a nucleus, and so we must have a different cross-section for each of these possible reactions. The reaction cross-section then becomes a purely fictitious concept introduced to provide a "picture" of what happens and to enable one to do quantitative calculations of relative reaction rates.

For any particular reaction (e.g., radiative capture), we again visualize each of the nuclei as presenting a certain (flat) target area to the incoming neutron. If the neutron strikes this target area, then the particular reaction being considered will occur. If it does not, then the reaction will not occur. Because the cross-section is associated with each individual nucleus, it is known as the ***microscopic*** cross-section. The values of the appropriate microscopic cross-sections for different target nuclei at various neutron energies have to be determined by experimental measurements of the reaction rates for the different types of reaction.

Microscopic cross-section

Each of the target nuclei can be visualized as having a number of different cross-sections, each proportional to the probability of a particular reaction (for example, elastic scattering, radiative capture or fission). The relative probabilities of a thermal neutron undergoing elastic scattering and radiative capture when it encounters a nucleus of U-238, for example, are directly proportional to the two cross-sectional areas illustrated in Figure 3.1.

Notes & References

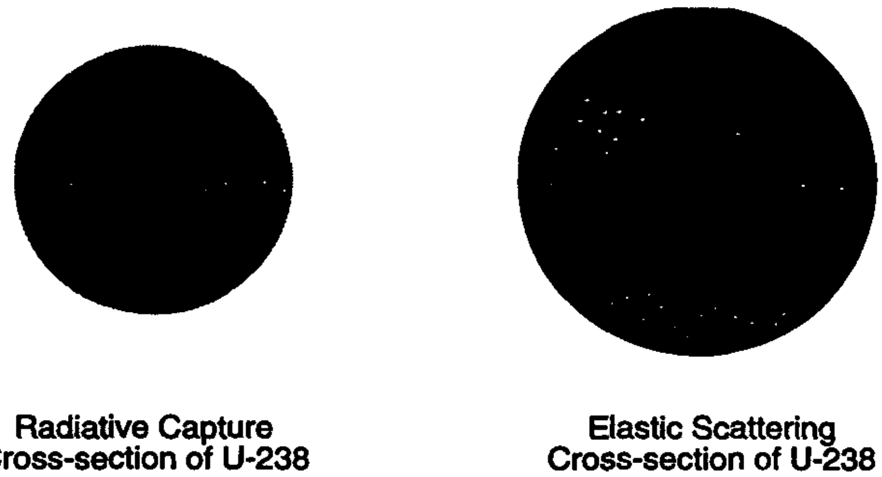


Figure 3.1 Cross-sections for U-238

In the example given, the cross-sectional area for elastic scattering is a factor of about 3.7 times greater than the radiative capture cross-section, since a thermal neutron encountering a U-238 nucleus is 3.7 times more likely to undergo elastic scattering than it is to undergo radiative capture. For neutrons with a different kinetic energy, the cross-sectional areas for both processes would be different, since the probabilities of the reactions occurring vary with the neutron energy (or speed).

Because the neutron cross-sections of the nuclei are so very tiny, they are specified in terms of a special unit, called the ***barn*** (abbreviated as b):

Barn (definition)

$$1\ barn = 10^{-24}\ cm^2$$

As an example, the cross-section of a nucleus of cadmium for radiative capture of a thermal neutron is 2.45×10^{-21} cm^2, or 2450 barns. This is about 1,800 times larger than the physical cross-sectional area of the nucleus (πr^2), which is about 1.4 barns. (Most nuclei have physical cross-sections in the order of 1 barn). On the other hand, many of the cross-sections of nuclei for certain reactions are much smaller than the physical cross-sectional area of the nucleus.

NOTE: You may wonder why the barn is defined in terms of cm^2 rather than m^2, as would be appropriate in the SI convention. The use of centimetres rather than meters is so well established in textbooks and other material that a change to SI units would cause confusion.

3.4 MICROSCOPIC CROSS-SECTIONS OF THE FISSILE ISOTOPES

The Greek letter σ (sigma) is used as the symbol for the microscopic cross-section, and the various possible reactions are specified by adding the appropriate suffix shown below:

σ_f	=	fission cross-section
σ_a	=	absorption cross-section
$\sigma_{n,\gamma}$	=	radiative capture cross-section
σ_i	=	inelastic scattering cross-section
σ_e	=	elastic scattering cross-section
σ_s	=	total scattering cross-section

For thermal neutrons, σ_a is usually just equal to the radiative capture cross-section, that is, $\sigma_{n,\gamma}$, since this is generally the only reaction that takes place when a neutron is absorbed. In those few cases where fission is also possible, (that is, $\sigma_f \neq 0$), σ_a would include σ_f and $\sigma_{n,\gamma}$, since a neutron is absorbed in both cases:

$$\sigma_a = \sigma_f + \sigma_{n,\gamma} \qquad (3.1)$$

The total scattering cross-section σ_s is the sum of the elastic and inelastic scattering cross-sections. For thermal neutrons, $\sigma_i = 0$ so that σ_s is simply equal to σ_e.

Cross-sections depend very much on neutron energy. Generally speaking, they are a lot larger at low energies than at high energies. For example, the fission cross-section σ_f for U-235 for neutrons of thermal energy is 580 b, whereas it is only just over 1 b at 1 MeV. In other words, fission of U-235 is about 500 times more likely for thermal neutrons than for fast neutrons. This is why it is desirable to have a ***moderator*** in a nuclear reactor.

For your interest, Table 3.1 lists the thermal neutron cross-sections of fuel nuclei. It is instructive to have a look at these numbers and see what we can make of them.

Notes & References

Thermal cross-section values

Table 3.1

Thermal neutron cross sections of fuel nuclei (in barns)

	σ_f	$\sigma_{n,\gamma}$	σ_a	σ_s	n	σ_f/σ_a (%)
U-233	530.6	47.0	577.6	10.7	2.487	92
U-235	580.2	98.3	678.5	17.6	2.430	86
U-238	0	2.71	2.71	~10	0	
Nat. U	4.18	3.40	7.58	~10		55
Pu-239	741.6	271.3	1012.9	8.5	2.890	73
Pu-241	1007.3	368.1	1375.4	12.0	2.934	73

Only 86% of the thermal neutrons absorbed by U-235 cause fission. You can see that this is just the fraction σ_f/σ_a. Note also that U-233 gives the greatest percentage of fission per neutron absorbed ($\sigma_f / \sigma_a = 92\%$).

To make the significance of the numbers easier to visualize, the cross-sections for U-235 and natural uranium are illustrated in the form of pie charts in Figure 3.2 below. The relative probabilities of the various reactions are proportional to the areas of the corresponding slices of the pie.

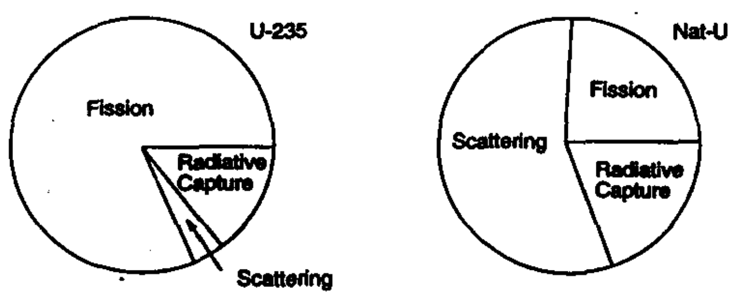


Figure 3.2: Thermal neutron cross-sections of U-235 and Nat-U

The "cross-sections" for natural uranium were obtained by taking a weighted mean of those for U-238 (99.28%) and U-235 (0.72%). Thus, the absorption cross-section is calculated as

$$\sigma_a\,(nat - U) = \frac{(99.28 \times 2.71) + (0.72 \times 678.5)}{100} = 7.58\ b$$

The values for natural uranium are really those that would apply to a fictitious "natural uranium nucleus", whose properties are obtained by a suitable weighting of those of the two uranium isotopes. The absorption cross-section of natural uranium, $\sigma_a = 7.58$ b, is, of course, equal to the sum of its fission and radiative absorption cross-sections ($\sigma_f = 4.18$ b, $\sigma_{a,\gamma} = 3.40$ b). This means that out of every 758 thermal neutrons absorbed in natural uranium, 418 will cause fission (in U-235) and the remaining 340 will undergo radiative capture (in either U-235 or U-238).

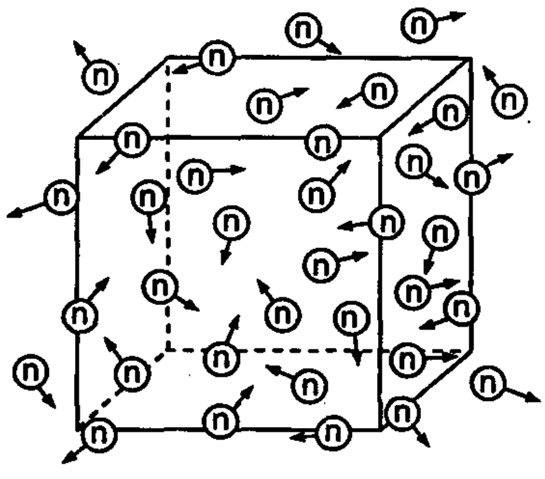
Since the 418 fissions in the U-235 will, on average, give rise to 2.43 fission neutrons, they will generate a total of $418 \times 2.43 =$ 1,016 new neutrons. This means that for every thermal neutron absorbed in natural uranium, we will get back an average of 1016/758 = 1.34 new ones. In the CANDU reactor, where relatively few neutrons are lost because the absorption cross-sections of the moderator and the core materials are low, the factor of 1.34 is more than adequate to maintain the chain reaction.

Notes & References

3.5 THE MACROSCOPIC CROSS-SECTION

The microscopic cross-sections of a given nucleus tell us the relative probabilities of a neutron inducing each of the possible reactions when it encounters the nucleus. To calculate the relative rates at which reactions take place in each material present in the reactor, we also need to take account of the relative numbers of atoms of these materials present in a given volume. To do this, we introduce a quantity known as the ***macroscopic*** cross-section, which combines the microscopic cross-section of an individual nucleus of a given material with the ***number density*** (that is, the number of atoms per unit volume) of the material in the region considered.

To explore the idea of a macroscopic cross-section, let's consider a 1 cm^3 cube of the reactor, with neutrons zipping around in it. We will try to work out what factors determine the rate neutrons are absorbed by the nuclei of a particular material present in this cube.



We start by defining the following quantities:

N = number of nuclei of the material under consideration in the 1 cm^3 volume

n = number of neutrons in the 1 cm^3 volume *(the neutron density)*

v = speed at which the neutrons travel around in the volume *(assumed to be the same for all of them)*

σ_a = microscopic absorption cross-section for the material (cm^2)

The absorption rate of neutrons by the nuclei of the material in the unit volume is given by the expression

Reaction rate per unit volume

$$R = n\mathrm{v}\, N\sigma_a \tag{3.2}$$

The proportionality of the absorption rate to the four quantities in the equation seems a reasonable one because:

a) the larger n is, the more neutrons will make collisions;

b) the larger their velocity, the more nuclei they will hit in a certain time;

c) the larger the number of target nuclei (N), the more will be hit; and

d) the larger the cross-section, the greater the probability of getting a hit.

The equation above is quite general. We could, for example, use it to calculate the ***fission rate*** per unit volume by substituting σ_f for σ_a. The quantities N and σ (for any reaction we are considering) are both characteristic of the target material. They are therefore often combined to give what is known as the ***macroscopic cross-section***, that is,

Macroscopic cross-section

$$\textit{macroscopic cross-section} \qquad \Sigma = N\sigma \qquad (3.3)$$

(Σ is the Greek capital σ). Note the distinction: the microscopic cross-section is related to the probability that a neutron encountering an ***individual nucleus*** will cause the particular reaction, while the macroscopic cross-section ***also*** takes account of the ***number density*** of the material considered.

Since σ is given in cm^2, and N in cm^{-3}, the ***units*** of Σ are $cm^2 \times cm^{-3}$, or cm^{-1}. As an illustration, let's work out the macroscopic absorption cross-section Σ_a for natural uranium. From Table 3.1, we know that the microscopic cross-section σ_a is equal to 7.58b. The other quantity we require is the number of uranium nuclei (that is, atoms) in one cm^3 of the material. This is given by the expression

$$N = \frac{0.6022 \times 10^{24}}{A} \times D \qquad (3.4)$$

where D is the density of the substance (in g/cm^3) and A is its atomic mass (as given, for example, in the Chart of Nuclides).

Using this expression, the macroscopic cross-section, Σ, is

$$\Sigma = \frac{0.6022 \times 10^{24}}{A} \times D \times \sigma \qquad (3.5)$$

The number of nuclei/cm³ for natural uranium (which is almost entirely U-238 and has a density of approximately 19.0 g/cm³) is therefore

$$N = \frac{0.6022 \times 10^{24}}{238} \times 19.0 = 4.8 \times 10^{22} = 0.048 \times 10^{24}$$

The macroscopic absorption cross-section is

$$\begin{aligned}\Sigma &= N\sigma_a \\ &= 0.048 \times 10^{24} \frac{1}{\text{cm}^3} \times 7.58 \times 10^{-24} (\text{cm}^2) \\ &= 0.36\ \text{cm}^{-1}\end{aligned}$$

We can do a similar calculation for uranium oxide fuel of the type used in a CANDU reactor. The density of the UO_2 is 10.8 g/cm³ and the A value used in equation (3.5) has to be adjusted to [238 + (2 × 16)] = 270 to allow for the fact that each molecule of UO_2 contains one uranium atom plus two oxygen atoms. With these changes, the value of Σ_a becomes 0.18 cm^{-1} and the average distance travelled by a thermal neutron in the oxide fuel becomes 5.5 cm (about 5 pellet diameters).

Absorption mean free path

It is more difficult to visualize the physical significance of the macroscopic cross-section with its rather odd units of cm^{-1} than it is for the microscopic cross-section, which is simply an area (although a somewhat artificial one). It can be shown, however, that the quantity $1/\Sigma_a$, which has the dimensions of a distance, does have an easily visualized meaning; it equals the ***average distance that a neutron will travel before being absorbed*** by the material, known as ***the absorption mean free path***. Thermal neutrons zipping around in a block of natural uranium, for instance, will travel an average distance of 1/0.36 = 2.8 cm before they are absorbed. Similarly, the inverse of the macroscopic scattering cross-section, $1/\Sigma_s$, is equal to the average distance travelled by a neutron between scattering collisions.

Notes & References

We can, of course, define individual macroscopic cross-sections for absorption, fission, radiative capture, scattering and so on. In each case, the ***reaction rate*** per unit volume, or the number of reactions per second of the particular type for which the microscopic cross-section is equal to σ, is given by

$$R = nvN\sigma = nv\Sigma \tag{3.6}$$

Appendix A gives the values of Σ_a for all of the elements and for light and heavy water. The cross-sections apply to ***thermal neutrons***. This table is included for interest's sake only, but it does identify the materials which have high neutron capture cross-sections and which don't.

3.6 NEUTRON FLUX

Equation (3.2) can be written in a slightly different form as

$$R = \phi\Sigma \tag{3.7}$$

where we have taken the product of the neutron density n and the neutron speed v as equal to a new quantity ϕ (the Greek letter "phi"), known as the ***neutron flux***, that is,

Neutron flux

$$\phi = nv \tag{3.8}$$

In physical terms, the quantity ϕ is the total distance travelled in one second by all the neutrons in 1 cm^3, since it is obtained by multiplying the number of neutrons in that cm^3 by the speed each is travelling. The ***units*** of flux are:

$$\frac{neutrons}{cm^3} \times \frac{cm}{s} \; or\; neutrons / cm^2 s$$

Notes & References

Although the expression for neutron flux applies to any neutron energy, the application most commonly encountered is to thermal neutrons, where the product is known as the ***thermal neutron flux***.

3.7 NEUTRON FLUX AND REACTOR POWER

Reaction rate and power

To see the use of some of the concepts introduced in this section, let's work out the total power generated in a typical CANDU reactor with a given thermal flux level.

Assume a fairly typical average neutron density in the fuel of 330 million neutrons per unit volume, that is, $n = 3.3 \times 10^8\ cm^{-3}$. To find the thermal neutron flux, ϕ, we have to multiply the neutron density by the average speed of thermal neutrons, which is 2.2 km/s, or 2.2×10^5 cm/s (or 5,000 m.p.h., if you like to look at it that way). The average thermal flux in the fuel is then

$$\phi = nv = 3.3 \times 10^8 \times 2.2 \times 10^5 = 7.3 \times 10^{13}\ n/cm^2\ s$$

Since the reactor uses uranium oxide fuel, for which we have already worked out that Σ_a for thermal neutrons is equal to $0.18\ cm^{-1}$, the neutron absorption rate per cm^3 of the fuel is

$$\phi\Sigma_a = 7.3 \times 10^{13} \times 0.18 = 1.3 \times 10^{13}\ absorptions/s.$$

If the volume (V) occupied by the fuel in the reactor is 9.3 m^3 or $9.3 \times 10^6\ cm^3$, then the total neutron absorption rate in the entire system is

$$\phi\Sigma_a V = 1.3 \times 10^{13} \times 9.3 \times 10^6 = 1.21 \times 10^{20}\ absorptions/s.$$

Going back to Table 3.1, you can see that 4.18 in every 7.58 neutrons absorbed will cause fission. The overall *fission rate* is therefore

$$\frac{4.18}{7.58} \times 1.21 \times 10^{20} = 6.7 \times 10^{19} \text{ fissions / s}$$

We mentioned in Section 2.8.7 that 3.1 x 10^{10} fissions/s will produce a power of 1 watt. Therefore, for this reactor, the total power is

$$\frac{6.7 \times 10^{19}}{3.1 \times 10^{10}} = 2.15 \times 10^{9}\,\text{W} = 2150\text{MW}$$

The power based on the fission rate in the core is known as the ***neutron power***. The distinction between neutron power and ***thermal power*** will be discussed in Module 10.

3.8 THE VARIATION OF THE MICROSCOPIC CROSS-SECTIONS OF URANIUM ISOTOPES WITH NEUTRON ENERGY

We have mentioned that the cross-section of a nuclide for neutron interactions is dependent on the kinetic energy of the neutron. In general, the absorption cross-section is higher for low-energy neutrons than it is for high-energy ones because slowly travelling neutrons spend more time in the vicinity of the nucleus, and therefore have more chance to interact. In some cases, however, we find that the cross-section rises steeply at certain specific values of the neutron energy, so that one gets a series of very sharp peaks, as shown by the absorption cross-section of U-238 in Figure 3.3. The whole energy region between about 5 eV and 1 keV is filled by a succession of these peaks, which are known as ***resonance absorption peaks***. (There are actually many more peaks than are shown in the figure). The corresponding neutron energies are called ***resonance energies***.

Resonance absorption in U-238

The absorption cross-section at the middle of the peak is so high that any neutron of that precise energy which encounters the uranium fuel will almost certainly be absorbed.

Notes & References

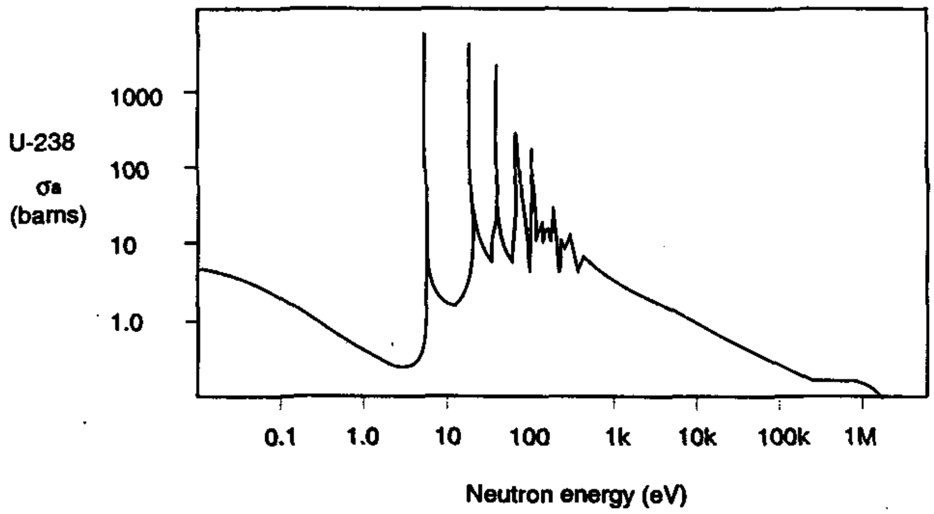


Figure 3.3 Variation of the absorption cross-section of U-238 with neutron energy

The absorption and fission cross-sections of U-235 also show a pronounced resonance structure as illustrated (for σ_a) in Figure 3.4. U-238 and the important isotope Pu-239, which is accumulated as a result of neutron capture in U-238, also show resonances in the energy region from a few eV up to several hundred eV, but Pu-239 also possesses a strong resonance just above thermal energies, at about 0.3 ev (Figure 12.7).

1/v absorbers

Although all cross-sections do not exhibit the same strong resonances as the uranium isotopes, all show some dependence on neutron energy. At low neutron energies, most absorption cross-sections are inversely proportional to the speed of the neutron, that is,

$$\sigma_a \propto \frac{1}{v}$$

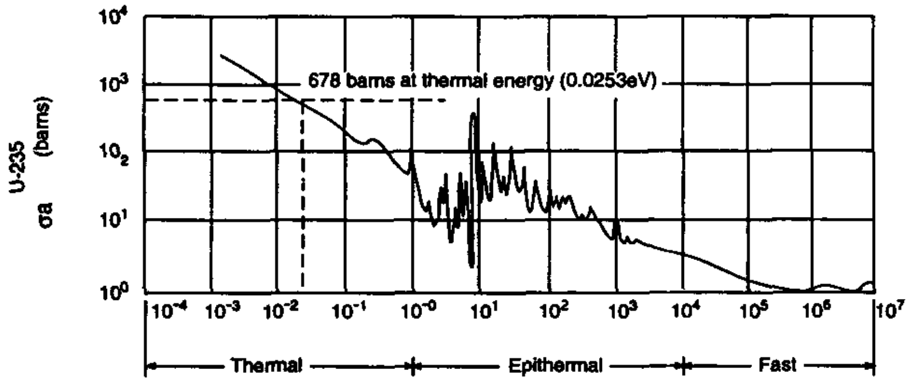


Figure 3.4 Variation of absorption cross-section of U-235 with neutron energy

Since the kinetic energy, E, is equal to ½ mv², v is proportional to √E, and the relation can also be written as

$$\sigma_a \propto \frac{1}{\sqrt{E}}$$

Variations from this type of dependence will be mentioned later in cases where they affect the behaviour of the reactor.

ASSIGNMENT

1. Given the microscopic cross-sections for U-235 and U-238 shown below (in barns), calculate the ***fission*** and ***absorption*** microscopic cross-sections for ***natural uranium***.

	σ_f	σ_γ	σ_a	σ_s
U-235	580.2	98.3	678.5	17.6
U-238	0	2.71	2.71	10

2. Using the cross-sections in the problem above, calculate the probability:

   a) that a thermal neutron absorbed by a U-235 nucleus will cause fission.

   b) that a thermal neutron will cause fission when it encounters a nucleus of U-235.

   c) that a thermal neutron travelling through a very large mass of natural uranium will eventually cause a fission.

3. Write down all the possible reactions that a thermal neutron can undergo while travelling through a mass of natural uranium.

4. The average number of neutrons per thermal fission in uranium is 2.43. If 100 thermal neutrons were absorbed in natural uranium, how many fast neutrons would be produced? Explain the significance of this number. (Use cross-sections calculated in problem 1).

5. At a certain stage of irradiation of a CANDU core, the proportions by weight of the three fissile isotopes present in the fuel are:

U-235	3.76 g/kgU
Pu-239	1.98 g/kgU
Pu-241	0.08 g/kgU

Calculate what fraction of the fissions are taking place in plutonium at this point. (Use the data in Table 3.1 and ignore differences in atomic mass between the 3 nuclides).